Vectors EX-1. 1

6) If $|\vec{a}| = 5$, (i) $|2\vec{a}|$ (ii) $|-\frac{1}{2}\vec{a}|$ (iii) $-\frac{1}{2}|\vec{a}|$

(i) $|2\vec{a}| = |2|\,|\vec{a}| = 2 \times 5 = 10$

(ii) $|-\frac{1}{2}\vec{a}| = |-\frac{1}{2}|\,|\vec{a}| = \frac{1}{2} \times 5 = \frac{5}{2}$

(iii) $-\frac{1}{2}|\vec{a}| = -\frac{1}{2} \times 5 = -\frac{5}{2}$

(7) $(\hat{i}+\hat{j}+\hat{k})p$ is a unit vector

$|\vec{a}|=1$

$|(\hat{i}+\hat{j}+\hat{k})p|=1$

$|p|\,|(\hat{i}+\hat{j}+\hat{k})|=1$

$|p|\left(\sqrt{1+1+1}\right)=1$

$|p|\left(\sqrt{3}\right)=1$

$|p|=\frac{1}{\sqrt{3}}$

$p=\pm\frac{1}{\sqrt{3}}$

$\hat{a}=\frac{\vec{a}}{|\vec{a}|}$

$\boxed{|\vec{a}|=1}$

(ii) $\vec{a} = -3\hat{i} - 2\hat{j} + 6\hat{k}$

$|\lambda \vec{a}| = 1$

$|\lambda||\vec{a}| = 1$

$|\vec{a}| = \sqrt{(-3)^2 + (-2)^2 + (6)^2}$

$= \sqrt{9 + 4 + 36}$

$|\vec{a}| = \sqrt{49} = 7$

$|\lambda| \cdot 7 = 1$

$\boxed{\lambda = \pm \frac{1}{7}}$

(9) (i) $2\hat{i}+3\hat{j}$ and $x\hat{i}+y\hat{j}$ are Equal.

$$2\hat{i}+3\hat{j} = x\hat{i}+y\hat{j}$$

$$x=2$$
$$y=3$$

$$a_1\hat{i}+b_1\hat{j}+c_1\hat{k}=a_2\hat{i}+b_2\hat{j}+c_2\hat{k}$$

$$a_1=a_2 \qquad c_1=c_2$$
$$b_1=b_2$$

(ii) $\vec{a} = x\hat{i} + 2\hat{j} - 3\hat{k}$ and $\vec{b} = 3\hat{i} - y\hat{j} + \hat{k}$ Equal vector

$x + y + z = ?$

$\vec{a} = \vec{b}$

$x\hat{i} + 2\hat{j} - z\hat{k} = 3\hat{i} - y\hat{j} + \hat{k}$

$x = 3 \quad y = -2 \quad z = -1$

$x + y + z = 3 - 2 - 1 = 3 - 3$

$= 0$ Ans

(9)(i) Let $\vec{a} = \hat{i} + 2\hat{j}$ and $\vec{b} = 2\hat{i} + \hat{j}$ $\quad |\vec{a}| = |\vec{b}|$

$|\vec{a}| = \sqrt{(1)^2 + (2)^2}$ $\quad |\vec{b}| = \sqrt{(2)^2 + (1)^2} = \sqrt{4+1} = \sqrt{5}$

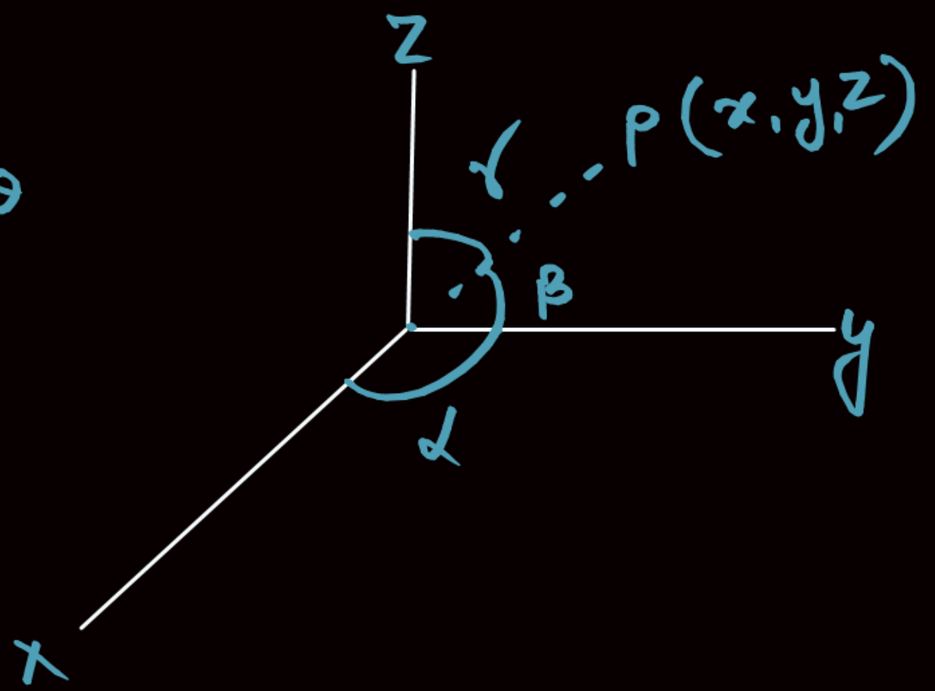
$|\vec{a}| = \sqrt{1+4} = \sqrt{5}$

$\sqrt{5} = \sqrt{5}$

$\alpha = \frac{\pi}{3}$ $\beta = \frac{\pi}{4}$ $\gamma = \theta$

$\cos\alpha = \frac{1}{2}$ $\cos\beta = \frac{1}{\sqrt{2}}$ $\gamma = \cos\theta$

$l = \frac{1}{2}$ $m = \frac{1}{\sqrt{2}}$ $n = \cos\theta$



Unit vector

$$\vec{a} = \frac{1}{2}\hat{i} + \frac{1}{\sqrt{2}}\hat{j} + \cos\theta\,\hat{k}$$

$$|\vec{a}| = \sqrt{\frac{1}{4} + \frac{1}{2} + \cos^2\theta}$$

$$|\vec{a}| = \sqrt{\frac{1+2}{4} + \cos^2\theta}$$

$$|\vec{a}| = \sqrt{\frac{3}{4} + \cos^2\theta}$$

$$1 = \frac{3}{4} + \cos^2\theta$$

$$\cos^2\theta = 1 - \frac{3}{4} = \frac{1}{4}$$

$$\cos^2\theta = \frac{1}{4}$$

$$\cos\theta = \frac{1}{2}$$

$$\boxed{\theta = \frac{\pi}{3}}$$